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# Modelling the variation of degree of saturation in a deformable unsaturated soil

D. GALLIPOLI,\* S. J. WHEELER† and M. KARSTUNEN†

An improved relationship for the variation of degree of saturation in an unsaturated soil is presented, incorporating the influence of changes of void ratio. When combined with an elasto-plastic stress–strain model, this is able to represent irreversible changes of degree of saturation and changes of degree of saturation caused by shearing. Experimental data from tests on compacted Speswhite kaolin are used to demonstrate the success of the proposed new expression for degree of saturation. The experimental data involve a wide variety of stress paths, including wetting, isotropic loading and unloading under constant suction, constant suction shearing, and constant water content shearing. Improved representation of the variation of degree of saturation has important consequences for numerical modelling of coupled flow-deformation problems, where the expression used for the degree of saturation can influence significantly the suction generated within the soil and hence the predicted stress–strain behaviour.

**KEYWORDS:** constitutive relations; partial saturation; suction

Nous présentons une relation améliorée pour la variation du degré de saturation dans un sol non saturé, en tenant compte de l'influence des changements de taux de pores. Combinée avec un modèle élasto-plastique contrainte-déformation, cette relation est capable de représenter les changements de degré de saturation irréversibles et les changements de degré de saturation causés par le cisaillement. Nous utilisons les données expérimentales tirées d'essais pratiqués sur du kaolin Speswhite compacté pour montrer le succès de notre nouvelle expression du degré de saturation. Les données expérimentales reposent sur une grande variété de chemins de contrainte, dont le mouillage, le chargement et déchargement isotropes sous succion constante, le cisaillement de succion constante et le cisaillement constant du contenu en eau. Une représentation améliorée de la variation du degré de saturation a des conséquences importantes sur la modélisation numérique de problèmes couplés d'écoulement et déformation, quand l'expression utilisée pour le degré de saturation peut influencer de manière significative la succion produite à l'intérieur du sol et, de là, le comportement prédit de contrainte-déformation.

## INTRODUCTION

Elasto-plastic stress–strain models for unsaturated soils, such as those proposed by Alonso *et al.* (1990) and Wheeler & Sivakumar (1995), are increasingly employed in numerical modelling of coupled flow-deformation problems involving unsaturated conditions. Typically, such stress–strain models relate the development of strains to the variation (and history of variation) of net stresses (the difference between total stresses and pore air pressure) and suction (the difference between pore air pressure and pore water pressure). Suction is therefore introduced as an additional stress variable over those required for saturated soils.

Existing unsaturated elasto-plastic stress–strain models tend not to provide information on the variation of degree of saturation,  $S_r$ , which is essential for coupled flow-deformation analyses. Numerical analyses performed to date with unsaturated elasto-plastic stress–strain models (e.g. Gatmiri *et al.*, 1995; Gens *et al.*, 1995; Thomas and He, 1998) have generally overcome this by assuming either a unique water-retention curve, relating  $S_r$  solely to suction,  $s$  (also known as a soil–water characteristic curve), or a unique 'state surface' expression, relating  $S_r$  to suction,  $s$ , and mean net stress,  $\bar{p}$ . Examples of the former type of expression are reviewed by Sillers *et al.* (2001), whereas examples of the latter are presented by Lloret & Alonso (1985).

In practice, the relationship between degree of saturation,

$S_r$ , and suction,  $s$ , for a given soil will be non-unique for at least two reasons:

- (a) In a deformable soil, variation of the void ratio produces changes in the dimensions of voids and of connecting passageways between voids, which cause corresponding changes in the water-retention curve.
- (b) Even in a rigid soil, the occurrence of 'hydraulic hysteresis' during inflow and outflow of water to individual voids means that retention curves followed during wetting and drying are different (Croney, 1952).

This paper suggests an improved form of expression for the variation of degree of saturation, accounting for the influence of changes of void ratio. The influence of hydraulic hysteresis is not included.

As changes of void ratio affect the water-retention curve, irreversible changes of degree of saturation will occur during loading and unloading of net stresses if there is an irreversible change of void ratio. An example of this is given in Fig. 1, which shows experimental data from an isotropic loading–unloading test on compacted Speswhite kaolin performed at a constant suction of 100 kPa by Zakaria (1995). During isotropic loading, when large plastic reductions in void ratio occurred, a significant increase in degree of saturation was observed, even though the suction was maintained constant. In contrast, during subsequent isotropic unloading, when only a very small elastic component of swelling occurred, the changes of degree of saturation were very modest. Similar irreversible changes of degree of saturation are reported by authors such as Josa (1988) and Romero (1999). The type of behaviour illustrated in Fig. 1 could not be represented by either a unique water-retention curve (which would predict no change of  $S_r$ ) or a unique state surface expression (which would predict reversible

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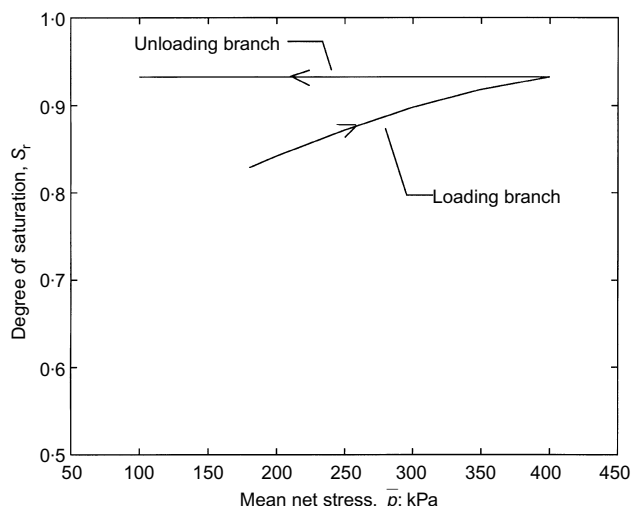


Fig. 1. Isotropic loading-unloading test at constant suction on compacted Speswhite kaolin (Zakaria, 1995)

changes of  $S_r$ ). Authors as early as Matyas & Radhakrishna (1968) emphasised that the assumption of a unique state surface for  $S_r$  was valid only for stress paths where net stresses were not decreased and suction was not increased.

Another important consequence of the influence of void ratio on the water-retention curve is that changes of  $S_r$  can be caused by the application of deviator stress  $q$  (even if suction and mean net stress remain constant), if shearing causes a change of void ratio. This is illustrated in Fig. 2, which shows experimental results from a triaxial shear test on compacted Speswhite kaolin performed by Sivakumar (1993), in which suction was maintained constant at 100 kPa and mean net stress was held constant at 100 kPa. A significant increase in degree of saturation was observed during shearing, attributable to the substantial decrease in void ratio that occurred. Either a unique water-retention curve or a unique state surface expression would predict no change of degree of saturation for the type of test illustrated in Fig. 2.

Wheeler (1996) presented an attempt to model irreversible changes of degree of saturation arising from the influence of changing void ratio, and the proposals presented in this paper can be viewed as an improvement and simplification

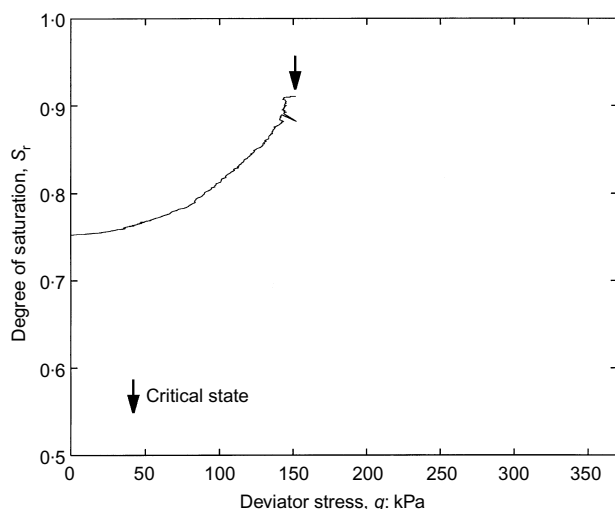


Fig. 2. Triaxial shear test at constant suction and constant mean stress on compacted Speswhite kaolin (Sivakumar, 1993)

of this earlier work. More recently, Vaunat *et al.* (2000) attempted to model the effects of both changing void ratio and hydraulic hysteresis on the variation of degree of saturation. The approaches proposed by Wheeler (1996), by Vaunat *et al.* (2000) and in the current paper can all be combined with existing stress-strain models (e.g. Alonso *et al.*, 1990; Wheeler & Sivakumar, 1995). A more difficult issue—that degree of saturation also appears to influence stress-strain behaviour—is starting to be tackled by authors such as Buisson & Wheeler (2000), Gallipoli *et al.* (2003) and Wheeler *et al.* (2003). Representation of this phenomenon requires a radical revision of existing stress-strain models, is relevant mainly to problems involving significant reversals or cycles of suction (where hydraulic hysteresis is important), and is outside the scope of this paper.

## PROPOSED RELATIONSHIP FOR $S_r$

### General proposal

The general hypothesis investigated here is that, in the absence of hydraulic hysteresis effects, there is, for a given soil, a unique relationship between degree of saturation,  $S_r$ , suction,  $s$ , and specific volume,  $v$ :

$$S_r = S_r(s, v) \quad (1)$$

Equation (1) describes a series of water-retention curves, of  $S_r$  plotted against  $s$ , each for a different value of  $v$ . As the specific volume decreases, the dimensions of voids and of connecting passageways between voids would be expected to decrease, so that a higher value of suction would be required to produce a given degree of saturation, resulting in a corresponding shift of the water-retention curve. If equation (1) is combined with a stress-strain model that introduces an elasto-plastic variation of specific volume  $v$ , then degree of saturation  $S_r$  is also predicted to vary in an elasto-plastic fashion.

Equation (1) involves a number of simplifying assumptions, including the following:

- Any influence of shear strains (which might also cause changes in the geometry of voids and of connecting passageways between voids) on degree of saturation is ignored.
- No distinction is made between the influence of elastic volumetric strains and plastic volumetric strains, whereas in practice these two components of volumetric strain are caused by different physical processes and, in consequence, they might cause different changes in void geometries.
- As already mentioned, the influence of hydraulic hysteresis is ignored.

Figure 3 shows experimental data from tests on compacted Speswhite kaolin performed by Sivakumar (1993) which provide support for the general proposal of equation (1). The dashed lines in Fig. 3 represent the experimental variation of  $S_r$  on isotropic normal compression lines at three different values of suction (100, 200 and 300 kPa). Each of these dashed lines was taken as the average from the post-yield sections of several isotropic loading tests performed at the given value of suction. Also shown in Fig. 3 are individual experimental data points corresponding to the final critical-state values of  $S_r$  reached during various forms of constant suction triaxial shear test performed at the same three values of suction. It is important to note that significant changes in specific volume occur during constant suction shearing from an isotropic stress state to a critical state, and therefore, for a given value of suction, critical-state data and isotropic normal compression data at the same value of  $v$  in Fig. 3 correspond to markedly different values of mean net stress,  $\bar{p}$ .

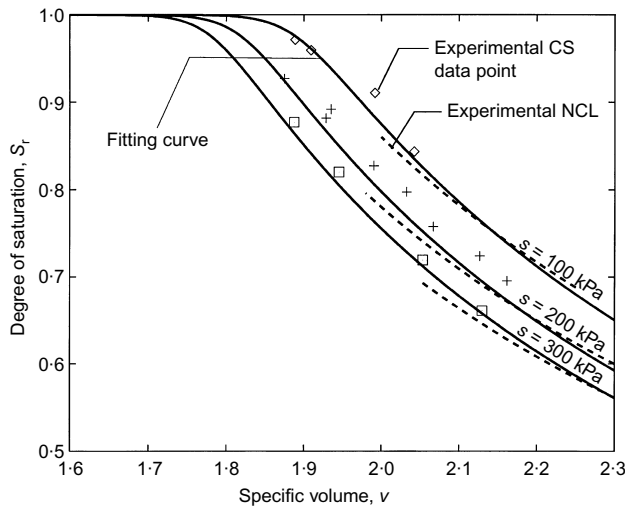


Fig. 3. Values of  $S_r$  on normal compression lines and at critical states for compacted Speswhite kaolin (experimental data from Sivakumar, 1993)

If equation (1) were rigorously correct, critical-state data points and isotropic normal compression line data at the same value of suction would form a single continuous line in Fig. 3. Inspection shows that this is approximately true, and the combined isotropic normal compression data and critical-state data can be satisfactorily represented by the three best-fit curves shown by the solid lines in the figure. It is worth noting that, whereas an isotropic normal compression state at given values of  $s$  and  $v$  and a critical state at the same values of  $s$  and  $v$  produce very similar values of degree of saturation,  $S_r$ , this is certainly not true of isotropic normal compression states and critical states when compared at the same values of  $s$  and  $\bar{p}$  (as is assumed in a state surface expression for  $S_r$ ).

Gallipoli (2000) also considered an alternative hypothesis to equation (1), that the variation of degree of saturation,  $S_r$ , was uniquely related to suction,  $s$ , and only the plastic components of volumetric strains (elastic volumetric strains were assumed to have no influence on  $S_r$ ). Using the same experimental data as shown in Fig. 3, he found that this alternative hypothesis provided a marginally better fit than equation (1) (the slight mismatch between isotropic normal compression data and critical-state data in Fig. 3 was reduced). The improvement was, however, relatively minor, and equation (1) has the advantage that it can be combined with any form of stress-strain constitutive model rather than being limited to use with an elasto-plastic model.

#### Explicit form of proposed relationship

An explicit form of equation is proposed for the variation of degree of saturation. The proposed equation tends to a simplified version of the water-retention curve expression of van Genuchten (1980) for the limiting case of an incompressible soil ( $v = \text{constant}$ ). The equation proposed by van Genuchten (1980) relates the volumetric water content to the suction, but for the case of an incompressible soil (as typically assumed by soil scientists) this equation can be rewritten in terms of degree of saturation rather than volumetric water content. By making the additional assumptions that the degree of saturation tends to unity at zero suction and to zero when suction tends to infinity (that is, the residual degree of saturation is zero), a simplified version of the van Genuchten (1980) expression is

$$S_r = \left( \frac{1}{1 + (\alpha s)^n} \right)^m \quad (2)$$

where  $m$ ,  $n$  and  $\alpha$  are soil constants.

Within geotechnical engineering changes of soil volume are highly significant, and the objective was therefore to explore whether equation (2) could be modified to take account of volume changes, by making one or more of the parameters  $m$ ,  $n$  and  $\alpha$  dependent on specific volume,  $v$ . Gallipoli (2000) analysed the experimental data for compacted Speswhite kaolin shown in Fig. 3 and found that these could be well matched with constant values for  $m$  and  $n$ , but for  $\alpha$  taken as the following function of specific volume:

$$\alpha = \phi(v - 1)^\psi \quad (3)$$

where  $\phi$  and  $\psi$  are soil constants. The proposed form of relationship between degree of saturation, suction and specific volume is therefore

$$S_r = \left\{ \frac{1}{1 + [\phi(v - 1)^\psi s]^n} \right\}^m \quad (4)$$

where  $m$ ,  $n$ ,  $\phi$  and  $\psi$  are soil constants.

Values of  $m = 0.03586$ ,  $n = 3.746$ ,  $\phi = 0.02691 \text{ kPa}^{-1}$  and  $\psi = 8.433$  were obtained for compacted Speswhite kaolin by performing a three-dimensional least-squares fit, in  $S_r : s : v$  space, of equation (4) to the isotropic normal compression data and critical-state data of Sivakumar (1993) shown in Fig. 3. In order to assign the same weight to critical-state and isotropic normal compression data, the fitting exercise involved sampling the normal compression lines at a number of points equal to the number of critical-state data points at the same suction.

Figure 4 shows a three-dimensional plot of  $S_r$  against  $s$  and  $v$  predicted by equation (4), using the parameter values appropriate to compacted Speswhite kaolin. The same information is shown in Fig. 5 as a series of water-retention curves at different values of  $v$ . Inspection of Figs 4 and 5 shows that variation of specific volume,  $v$ , can be as significant as variation of suction,  $s$ , in terms of the change of degree of saturation produced.

Analysis of data from other soils, in the future, may indicate that alternative expressions to equation (4) have

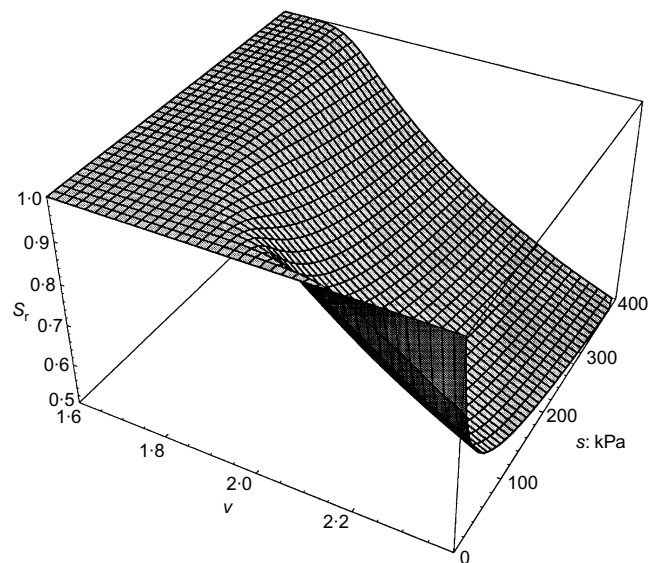


Fig. 4. Surface fitting experimental normal compression lines and critical-state data points in  $(S_r, s, v)$  space

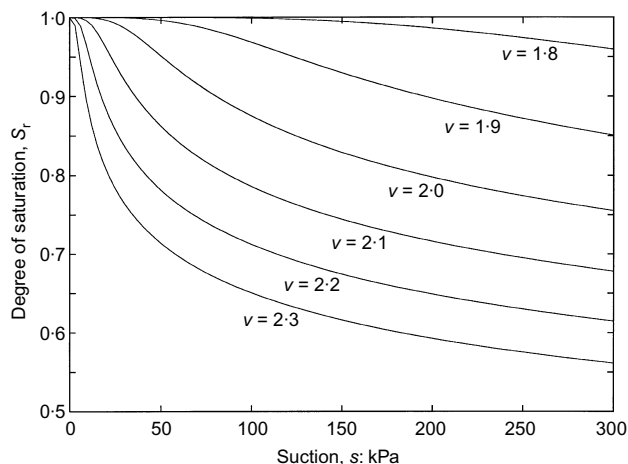


Fig. 5. Predicted water-retention curves at different values of  $\nu$

advantages. The general form of equation given in equation (1) should be considered as the main proposal of this paper, rather than the specific form presented in equation (4).

#### MODEL PREDICTIONS AND COMPARISON WITH EXPERIMENTAL RESULTS

Predictions of the variation of degree of saturation are compared against experimental observations from tests on compacted Speswhite kaolin performed by Sivakumar (1993) and Zakaria (1995). All samples were statically compacted under a vertical stress of 400 kPa at a compaction water content of 25% (4% dry of Proctor optimum). Each test commenced with wetting under a low applied net stress from the as-compacted suction of about 700 kPa to a suction of zero, 100, 200 or 300 kPa. This was typically followed by isotropic loading at a constant value of suction and then some form of suction-controlled shearing to a critical state (see Wheeler & Sivakumar, 1995, for fuller details). In the majority of tests the suction was either decreasing or held constant in every test stage, and—in the absence of reversals or cycles of suction—there was therefore no influence of hydraulic hysteresis.

Theoretical predictions of the variation of degree of saturation were calculated from equation (4), using the values of  $m$ ,  $n$ ,  $\phi$  and  $\psi$  for compacted Speswhite kaolin presented in the previous section. Values of specific volume,  $\nu$ , were calculated from the elasto-plastic model of Wheeler & Sivakumar (1995), using model parameter values suggested by these authors for compacted Speswhite kaolin.

Alternative theoretical predictions were also calculated, by using the conventional state surface expression for degree of saturation suggested by Lloret & Alonso (1985):

$$S_r = a - \tanh(bs)(c + d\bar{p}) \quad (5)$$

A value of 1 was used for the parameter  $a$  (in order to predict full saturation at zero suction), and the values of  $b$ ,  $c$  and  $d$  for compacted Speswhite kaolin were determined by fitting the three experimental isotropic normal compression lines at constant suctions of 100, 200 and 300 kPa of Sivakumar (1993). This resulted in  $b = 0.00534 \text{ kPa}^{-1}$ ,  $c = 0.554$ , and  $d = -0.00107 \text{ kPa}^{-1}$ . Given  $a = 1$ , equation (5) requires one less soil constant than equation (4).

For each of the two methods of predicting  $S_r$  (the proposed new relationship of equation (4) and the conventional state surface expression of equation (5)) the challenge was whether the variation of  $S_r$  could be successfully

predicted throughout the full range of different stress paths followed in the various stages of the different tests.

#### Isotropic normal compression at constant suction

Figure 6 shows experimental and predicted variations of degree of saturation during the post-yield sections of isotropic loading at three different values of suction (100, 200 and 300 kPa). The experimental curves shown in the figure were used in selecting the values of the various parameters

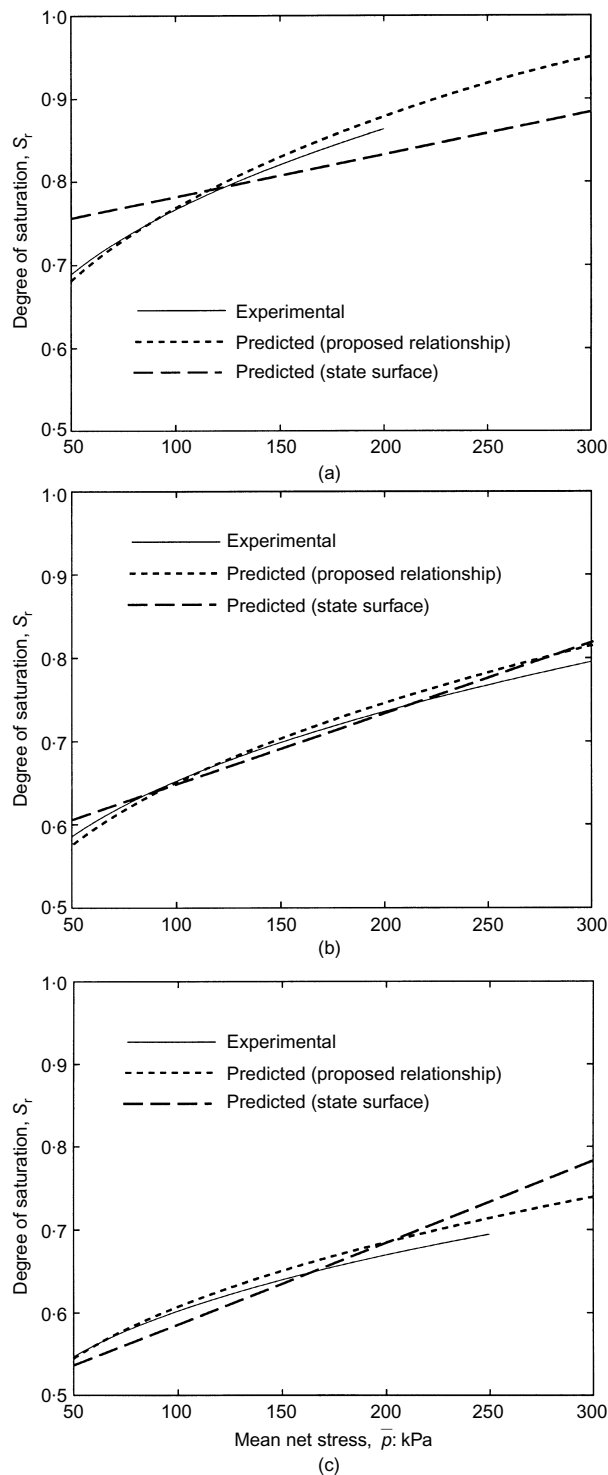


Fig. 6. Experimental and predicted values of  $S_r$  on isotropic normal compression lines at: (a)  $s = 100 \text{ kPa}$ ; (b)  $s = 200 \text{ kPa}$ ; (c)  $s = 300 \text{ kPa}$  (experimental data from Sivakumar, 1993)

in equations (4) and (5), and they do not therefore contribute to any independent validation of the two predictive methods.

Inspection of Fig. 6 shows that the proposed new relationship for  $S_r$  of equation (4) provides a good match to the experimental curves at all three values of suction. The conventional state surface expression for  $S_r$  of equation (5) provides a reasonable match at  $s = 200$  kPa and  $s = 300$  kPa, but the match is less good at  $s = 100$  kPa. This simply indicates that an alternative explicit form of state surface expression for  $S_r$  might provide a better match to the isotropic normal compression behaviour at all three values of suction.

#### Critical states

Figure 7 shows experimental and predicted values of  $S_r$  for critical states at three different values of suction. The proposed new relationship for  $S_r$  (equation (4)), when combined with the elasto-plastic model of Wheeler & Sivakumar (1995), provides a good match to the experimental data. In contrast, the state surface expression of equation (5) predicts exactly the same curves in Figs 6 and 7, and in consequence is unable to provide a good match to the experimentally observed critical-state values of  $S_r$ . Therefore, whereas the new relationship for  $S_r$ , which includes dependence on  $\nu$ , is able to match both isotropic normal compression and critical-state values of  $S_r$  with a single set of model parameter values, this is not possible with a conventional state surface expression for  $S_r$ .

#### Isotropic loading–unloading at constant suction

Figure 8 shows experimental results and corresponding predictions for an isotropic loading–unloading test at a constant suction of 100 kPa (the same test as shown in Fig. 1). This laboratory test was performed by Zakaria (1995), whereas all the other experimental tests presented here, including those used in the selection of parameter values, were performed by Sivakumar (1993).

During isotropic loading the values of  $S_r$  are overpredicted by the proposed new relationship of equation (4). This indicates that, although Zakaria (1995) ostensibly used the same material and procedures as Sivakumar (1993), use of a different batch of Speswhite kaolin, different equipment and a different operator resulted in slightly different values of  $S_r$  (compare Fig. 8 with Fig. 6(a)). The excellent match provided by the conventional state surface expression of equation (5) in predicting the variation of  $S_r$  during the isotropic loading stage of Zakaria's test (see Fig. 8) is largely fortuitous, with the difficulty of matching equation (5) to Sivakumar's isotropic compression data at three different values of suction (see Fig. 6) being coincidentally offset at  $s = 100$  kPa by the difference between Sivakumar's and Zakaria's data.

More significant is the predicted variation of  $S_r$  during unloading shown in Fig. 8. Whereas the conventional state surface expression of equation (5) predicts entirely reversible behaviour, the new relationship of equation (4) correctly predicts very little change of  $S_r$  during isotropic unloading (compared with the significant increase of  $S_r$  during previous loading).

#### Shearing at constant suction

Figure 9 shows the variation of  $S_r$  during a triaxial shearing stage performed by Sivakumar (1993), in which both suction,  $s$ , and mean net stress,  $\bar{p}$ , were maintained constant at 100 kPa (the same test as shown in Fig. 2). Prior to shearing, the sample had been isotropically compressed to

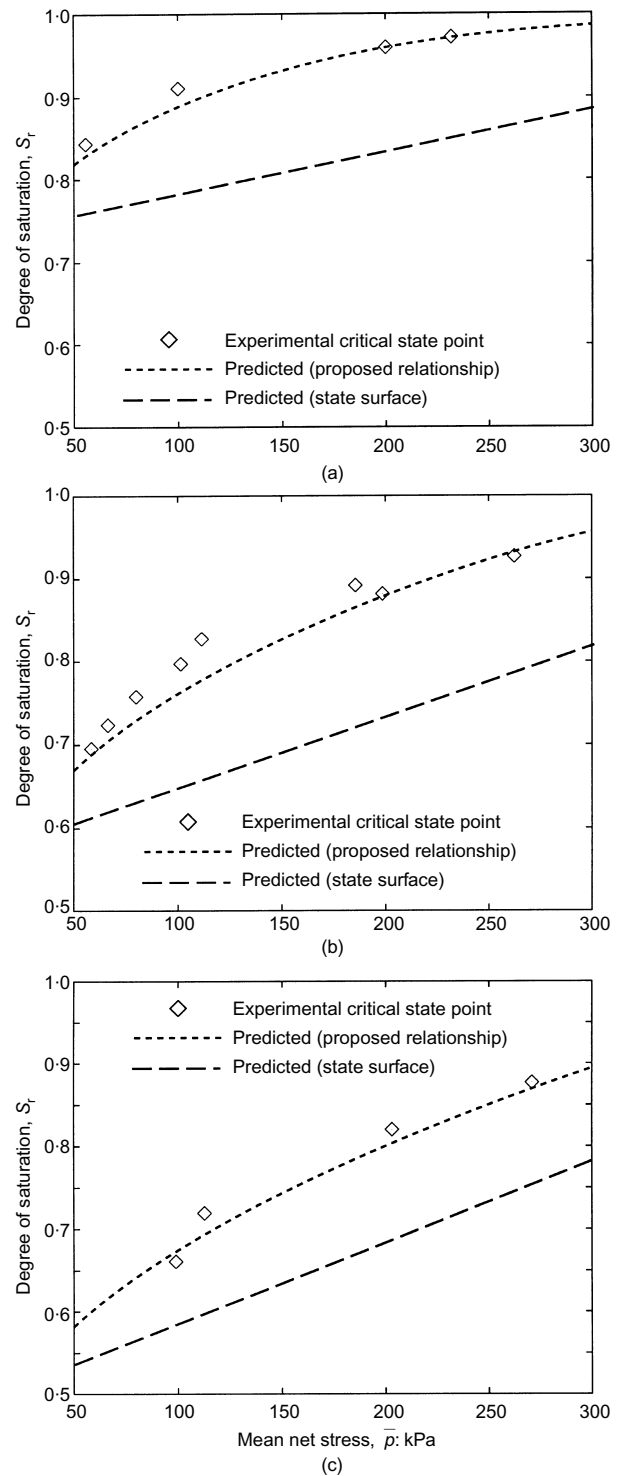


Fig. 7. Experimental and predicted values of  $S_r$  at critical states at: (a)  $s = 100$  kPa; (b)  $s = 200$  kPa; (c)  $s = 300$  kPa (experimental data from Sivakumar, 1993)

a state on the normal compression line for  $s = 100$  kPa. The experimental results show a significant increase in degree of saturation during shearing. This increase of  $S_r$  was produced by the combination of an inflow of water to the sample and a decrease in the total void volume as the sample compressed during shearing. The increase of  $S_r$  can be attributed to flooding of additional voids with water, as the void sizes decreased during compression and a number of voids reduced in size to below a critical dimension corresponding to flooding at the applied value of suction. For a soil with a

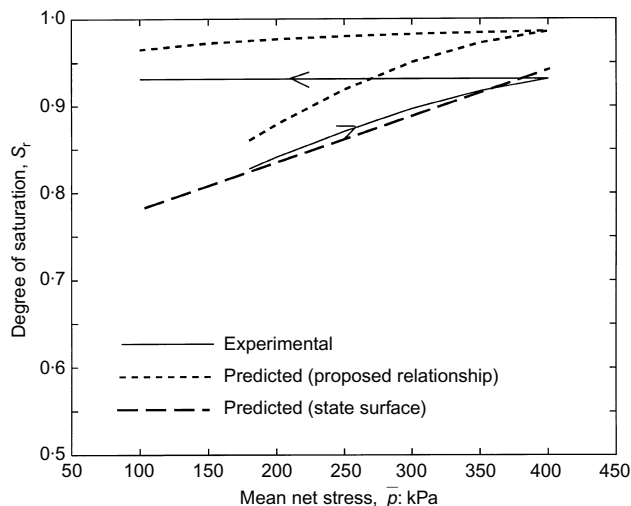


Fig. 8. Experimental and predicted variation of  $S_r$  during isotropic loading–unloading at constant suction ( $s = 100$  kPa) (experimental data from Zakaria, 1995)

bimodal pore size distribution, such as these samples compacted dry of optimum, the increase of  $S_r$  may have been accentuated by dilation of small water-filled microvoids within relatively dense soil packets (even though the more open macrostructure of the soil was compressing during shearing).

Figure 9 shows that the proposed new relationship for  $S_r$  provides a good match to the observed increase of  $S_r$  during shearing. In this simulation the elasto-plastic stress–strain model of Wheeler & Sivakumar (1995) predicts that the normally consolidated soil is compressing plastically throughout shearing, and the consequent reduction of  $\nu$  results in a significant increase of  $S_r$  predicted by equation (4). In contrast, the conventional state surface expression of equation (5) suggests no change of  $S_r$  during this type of shearing, where both  $s$  and  $\bar{p}$  are held constant. Gallipoli (2000) presented simulations of two other constant  $s$ /constant  $\bar{p}$  shear tests performed by Sivakumar (1993). A similar pattern of results was observed.

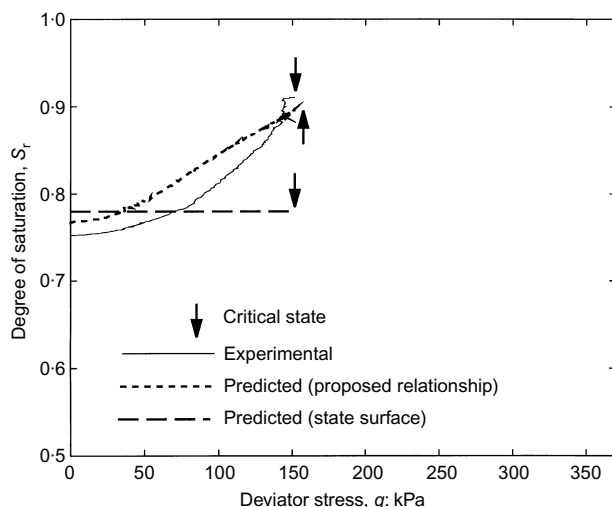


Fig. 9. Experimental and predicted variation of  $S_r$  during triaxial shearing at constant suction ( $s = 100$  kPa) and constant mean net stress (experimental data from Sivakumar, 1993)

#### Wetting under an isotropic stress state

Figure 10 shows experimental and predicted values of  $S_r$  during wetting from the as-compacted suction of 700 kPa under an isotropic stress state with  $\bar{p} = 50$  kPa. Experimental values of  $S_r$  were measured by Sivakumar (1993) at the start of wetting and on subsequent wetting to four different values of suction (300 kPa, 200 kPa, 100 kPa and zero). Each experimental data point represents the average of several tests. Samples brought to zero suction were forcibly saturated by flushing with water (see Wheeler & Sivakumar, 1995).

Figure 10 shows that equation (4) provides a remarkably accurate match to the observed variation of  $S_r$ , by incorporating the influence of the complex variation of  $\nu$  during wetting (elastic swelling during the early part of wetting, and then a change to plastic collapse compression at lower values of suction). Predictions from the conventional state surface expression of equation (5) are less accurate. In particular, values of  $S_r$  are underestimated at higher values of suction, when the stress state is still inside the yield surface.

#### Undrained shearing

Figure 11 shows the observed variation of suction during a triaxial shear test of Sivakumar (1993), performed with the water phase undrained and pore air pressure,  $u_a$ , maintained constant. Prior to shearing the sample had been isotropically compressed at a suction of 200 kPa to a mean net stress of 100 kPa. The experimental curve in Fig. 11 shows a significant increase of suction during undrained shearing. This is consistent with the fact that constant-suction shearing of this type of sample resulted in an inflow of water to the soil (see previous section).

Theoretical predictions of the variation of suction during undrained shearing were produced by coupled solution of the expression for  $S_r$  (either equation (4) or equation (5)) and the stress–strain model providing the variation of volumetric strain, after imposing the constraint that the product of  $S_r$  and void ratio,  $e$ , must remain constant. The elasto-plastic model of Wheeler & Sivakumar (1995) was used for the stress–strain behaviour. Wheeler & Sivakumar (1995) provided values of suction-dependent soil parameters for compacted Speswhite kaolin, for use within their model, only at discrete values of suction (zero, 100, 200 and

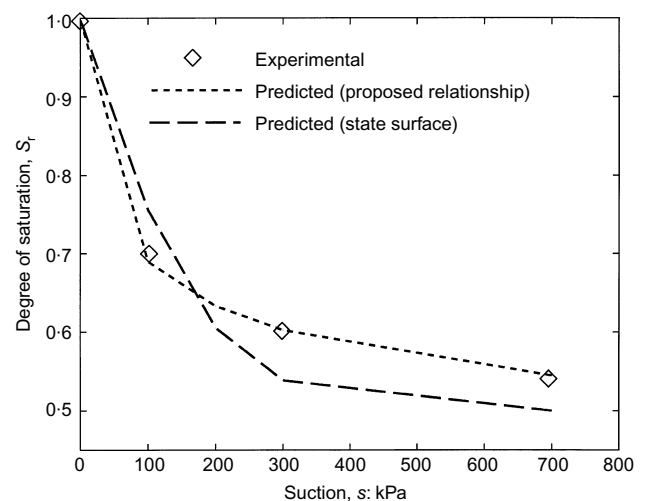


Fig. 10. Experimental and predicted variation of  $S_r$  during wetting under isotropic stress states ( $\bar{p} = 100$  kPa) (experimental data from Sivakumar, 1993)

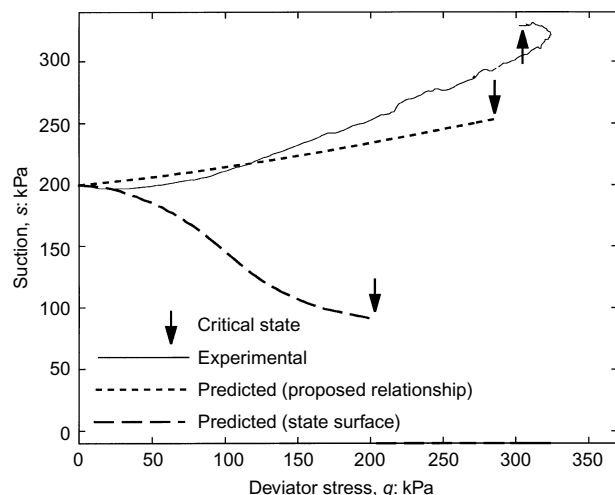


Fig. 11. Experimental and predicted variation of  $s$  during undrained triaxial shear starting from  $s = 200$  kPa,  $\bar{p} = 100$  kPa (experimental results from Sivakumar, 1993)

300 kPa). Linear interpolation of the soil parameters was therefore used for intermediate values of suction for the simulations shown in Fig. 11.

Inspection of Fig. 11 shows that the proposed new relationship for  $S_r$  of equation (4) correctly predicts an increase of suction during the undrained shearing stage, but the magnitude of suction increase is underestimated. This underestimation of the suction change can be attributed to the fact that hydraulic hysteresis was ignored. In this test, suction had previously been reduced from the as-compacted value of 700 kPa to a value of 200 kPa, but suction then increased again during the final undrained shear stage. This was therefore an example where the influence of hydraulic hysteresis was apparent. Fig. 11 shows that the conventional state surface expression for  $S_r$  of equation (5), when coupled with an elasto-plastic stress-strain model, incorrectly predicts a decrease in suction during the undrained shearing stage.

Gallipoli (2000) presented simulations of two other undrained shear tests performed by Sivakumar (1993). A similar pattern of results was observed, with the proposed new relationship of equation (4) providing substantially better predictions than the conventional state surface expression of equation (5), but with the observed increase of suction still being underestimated (thought to be due to the influence of hydraulic hysteresis).

## CONCLUSIONS

An improved form of relationship for the prediction of degree of saturation,  $S_r$ , is proposed, incorporating dependence on specific volume,  $v$ , as well as suction. When combined with an elasto-plastic stress-strain model, this type of relationship is able to predict irreversible changes of  $S_r$  and variation of  $S_r$  caused by shearing. A specific form for the proposed new relationship is suggested, based on the water-retention curve equation of van Genuchten (1980), but with the parameter  $\alpha$  expressed as a function of  $v$ . The proposed new relationship for  $S_r$  requires only one more soil constant than a conventional state surface expression for  $S_r$ .

Experimental data from tests on compacted Speswhite kaolin performed by Sivakumar (1993) and Zakaria (1995) were used to demonstrate the success of the proposed new expression for degree of saturation. The experimental data involved a wide variety of different stress paths, including isotropic loading and unloading at constant suction, wetting

under constant net stresses, and two different types of shearing to a critical state (constant suction shearing and constant water content shearing). The experimental tests were simulated by the proposed new relationship for degree of saturation, with the variation of specific volume provided by the elasto-plastic stress-strain model of Wheeler & Sivakumar (1995). Excellent agreement between model predictions and experimental results was observed for the full range of stress paths.

Corresponding simulations were also performed with a conventional state surface expression for  $S_r$ . These demonstrated that, whereas a state surface expression can match the observed variation of  $S_r$  for one particular type of stress path (by selection of suitable parameter values), it is incapable of representing the variation of  $S_r$  for the full range of stress paths with a single set of parameter values. Because of this, use of a state surface expression for  $S_r$  in numerical analyses should be limited ideally to situations where the stress path and stress history (including variation of suction) followed by all soil elements in a boundary value problem are identical to those followed in the laboratory tests used to determine parameter values. Clearly, this is impossible in practice, because different soil elements in a boundary value problem will all follow different stress paths.

Improved modelling of the variation of degree of saturation has important consequences for numerical analysis of boundary value problems involving coupling of hydraulic and mechanical behaviour. In particular, the values of suction predicted during undrained or partially drained loading (with respect to the water phase) are strongly influenced by the relationship used to represent the variation of  $S_r$  (as shown by the simulations of undrained triaxial shearing presented earlier). This influence on predicted suctions means that the stress-strain behaviour is affected, and hence predicted deformations and failure loads are influenced. Gallipoli *et al.* (2002) presented the results of numerical modelling of pressuremeter expansion in an unsaturated soil, using the elasto-plastic stress-strain model of Alonso *et al.* (1990) and either the proposed new relationship for  $S_r$  or a conventional state surface expression for  $S_r$ . Their results showed significant differences between the two sets of analyses, and thus provided an example of the practical significance of improved modelling of the variation of degree of saturation.

The proposed new relationship for  $S_r$  takes no account of any influence of hydraulic hysteresis during wetting and drying. It should therefore be used with caution during analysis of any problems involving significant reversals or cycles of suction.

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## NOTATION

$a, b, c, d$	parameters in state surface expression for $S_r$
$e$	void ratio
$m, n, \alpha$	parameters in simplified van Genuchten expression for $S_r$
$p$	mean total stress
$\bar{p}$	mean net stress ( $\bar{p} = p - u_a$ )
$q$	deviator stress
$s$	suction ( $s = u_a - u_w$ )
$S_r$	degree of saturation
$u_a$	pore air pressure
$u_w$	pore water pressure
$v$	specific volume ( $v = 1 + e$ )
$\phi, \Psi$	parameters in proposed expression for $S_r$



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